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تنظیم پذیری مدهای هایبرید در موجبرهای متاماده

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چکیده – در این مقاله، تنظیم پذیری ناحیه فرکانسی مدهای هایبرید در موجبرهای تخت متاماده-دی الکتریک با در نظر گرفتن مشخصه های ساختاری متفاوت مورد مطالعه قرار گرفته است.در این راستا، تاثیر مشخصه های ساختاری متامواد همچون، قدرت رزونانس مغناطیسی و میرایی مغناطیسی مورد بررسی قرار گرفت. مشاهده می شود که با کاهش قدرت رزونانس مغناطیسی، ناحیه ضریب شکست منفی کاهش پیدا می کند.

کلید واژه- متاماده، موجبرهای تخت متاماده-دی الکتریک، ضریب شکست منفی، مد هایبرید

Hybrid mode tunability in metamaterial waveguides

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Abstract- In this paper, the tunability of hybrid modes frequency region is studied in slab metamaterial-dielectric waveguides by considering different structural parameters. In this case, the effect of metamaterial structural parameters such as the magnetic oscillation strength and magnetic damping are studied. We can see that with a decrease in the magnetic oscillation strength, the negative refractive index region decreases.

Keywords: Metamaterial- Slab Metamaterial-dielectric waveguides-Negative refractive index-Hybrid mode

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1. Introduction

Metamaterials are artificial composite materials that have electromagnetic properties not attainable in nature, having negative refractive index (n) as well as applications in super-lenses [1].

One of the noticeable challenges in metamaterials is the tuning of frequency region in which negative n index exists. In order to find and tune the negative-index region in metamaterials, different methods can be used, one of which is working on the structural parameters of these materials.

We can study the behavior of n by investigating the electric and magnetic responses of metamaterials [2] and to tune the negative index region, the structural parameters should be considered. A set of essential parameters in this case are magnetic oscillation strength and damping constant of metamaterial structure [2, 3].

One of the structures based on metamaterials are metamaterial waveguides. To understand the properties of metamaterial waveguides, the dispersion equation should be solved numerically. Then, the behavior of modes in the waveguide can be studied. The tuning of n in metamaterials has effects on the modes behavior in metamaterial-dielectric waveguides, as well.

This paper intends to study the behavior of n in metamaterial structures and modes behavior in metamaterial-dielectric waveguides.

2. Backgrounds and Methods

To study the metamaterial waveguides, metamaterials with fishnet unit-cells are considered in this paper. The schematic diagram of metamaterial unit-cell structure is illustrated in Fig. 1 [3].

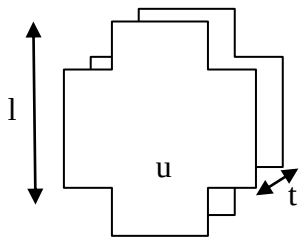


Figure 1.the fishnet unit-cell structure. u is the width, l is the length and t is the separation between two layers.

The behavior of n can be studied by

$$n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \qquad (1)$$

here, ε is electric permittivity and μ is magnetic permeability. ε of metamaterial can be obtained by Drude model [2]:

$$\varepsilon = \varepsilon_0(1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma_e)}), \qquad (2)$$

where, ω_p is plasma frequency, Γ_e is electric damping constant and ω is operating frequency. Each of these parameters has a certain value that respect the structural characteristics of a particular material. μ of a Fishnet metamaterial can be explained by Drude-Lorentz model as:

$$\mu = \mu_0(1 - \frac{F\omega_m{}^2}{\omega_m{}^2 - \omega_0^2 + i\Gamma_m\omega_m}) \qquad (3)$$

Where, ω_0 is resonance frequency, Γ_m is magnetic damping constant and F is magnetic oscillation strength. There are parameters such as Γ_m and F that are functions of the metamaterial structure. F follows the equation

$$F' = F\frac{L}{L + L_e}; F = \frac{ltu}{V_{uc}}. \qquad (4)$$

Where, L is Inductance, L_e is kinetic inductance and V_{uc} is volume of unit cell [3]. The magnetic damping constant can vary in the range of $10^{-5}\Gamma_e \prec \Gamma_m \leq \Gamma_e$ and F has values between 0.1-0.5 [3]. By tuning these structural parameters, we can tune the frequency region in which negative n exists.

We can then study metamaterial waveguides. Waveguides are structures constructed from core and cladding and can be made of different materials [4]. The applications of metamaterials in waveguides have attracted interest in recent years. In this paper, we study slab metamaterial-dielectric waveguide, Fig. 2.

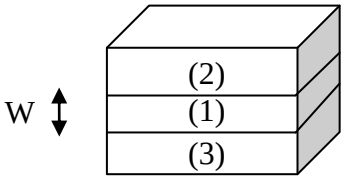


Figure 2. Slab waveguide consisting of three layers, layer (1) is the core and regions (2) and (3) are cladding. W is the width of core.

There are different modes in waveguides including transverse electromagnetic (TEM),

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transverse magnetic (TM) and transverse electric waves (TE) [5]. Studies show that these can have three modes behavior in metamaterial-dielectric waveguides as ordinary, surface and hybrid modes [6, 7], where hybrid modes are the combination of surface and ordinary modes.

To study the waveguide properties, we need to solve the dispersion relation of guide. For slab waveguide with metamaterial cladding, the dispersion relation is written as:

$$\frac{\varepsilon_1}{\gamma_1}\left(\frac{\varepsilon_2}{\gamma_2}+\frac{\varepsilon_3}{\gamma_3}\right)=-\left(\frac{\varepsilon_1^2}{\gamma_1^2}+\frac{\varepsilon_2\varepsilon_3}{\gamma_2\gamma_3}\right)\tanh\gamma_1 w. \quad (4)$$

Here, $\gamma_j=\sqrt{k_z^2-\omega^2\varepsilon_j\mu_j}$ is the complex wave number for the transverse component of the field. j=1, 2, 3 refers to the core and two cladding layers, respectively [7]. This dispersion equation should be solved numerically. We use numerical root-finding method based on Newton-Raphson method using FindRoot function in Mathematica software.

3. Results and discussion

In this section we study the behavior of n in metamaterials and modes behavior in slab metamaterial waveguide.

We consider $\mu_1=\mu_0$, $\omega_e=1.37\times10^{16}s^{-1}$, $F=0.5$, $\Gamma_m=\Gamma_e=2.73\times10^{13}s^{-1}$ and $\omega_0=0.2\omega_e$ [7]. We choose these parameters based on the previews studies in this field (Ref. [7]) to make the comparison possible between our and previews results.

Figure 3 illustrates the behavior of n in metamaterial structure as a function of frequency using Eq. (1).

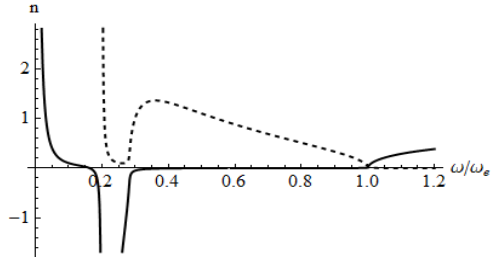


Figure 3. The plot of refractive index of the metamaterial as a function of frequency, the real parts (solid lines) and imaginary parts (dashed lines).

Figure 3 illustrates the behavior of real and imaginary parts of n as a function of frequency. We can see that the real part of n becomes negative in the frequency region $0.2\omega_e\le\omega\le0.3\omega_e$.

Now, to tune the n index in metamaterial structure, we consider different structural parameters as F and Γ_m. By examining different values for these parameters, we study the changes of negative n region in Figs. 4-5. These values are chosen by considering the permitted ranges for parameters so that we can see the critical points in the plots. We consider $F=0.1,\ 0.3$ and $\Gamma_m=0.1\Gamma_e,\ 0.001\Gamma_e$.

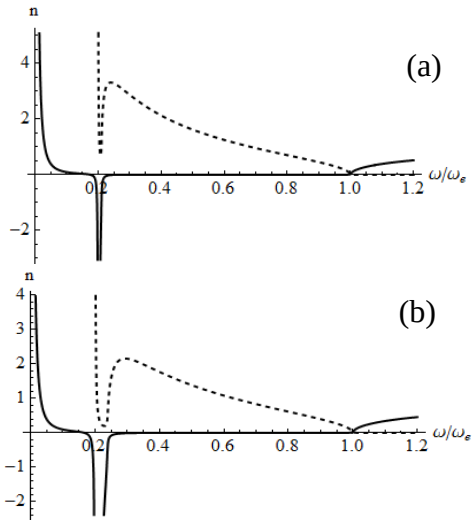
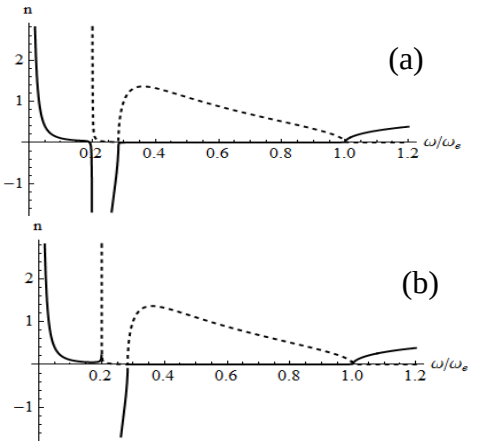


Figure 4. Plot of the real parts (solid lines) and imaginary parts (dashed lines) of the refractive index of the metamaterial, for (a) F=0.1 and (b) F=0.3.

Figure 4 show that when F decreases, the negative n region reduces and becomes narrower. Therefore, if we are interested in wider negative index regions, larger values of F are better.

The next effective parameter is magnetic damping constant whose effect on the negative-index region is illustrated in Fig. 5.



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Figure 5. Plot of real parts (solid line) and imaginary parts (dashed line) of metamaterial refractive index for (a) $\Gamma_m = 0.1\Gamma_e$ and (b) $\Gamma_m = 0.001\Gamma_e$

By comparing Figs. 3 and 5 (b), we can see that by choosing $\Gamma_m = 0.001\Gamma_e$, the imaginary part of n becomes zero in negative index region and the real part become distorted. This behavior shows that the lower limit for damping constant is $\Gamma_m = 0.001\Gamma_e$.

The modes behavior in the waveguide can be studied by considering the behavior of real to imaginary parts of the γ_1 [7] as Fig.6.

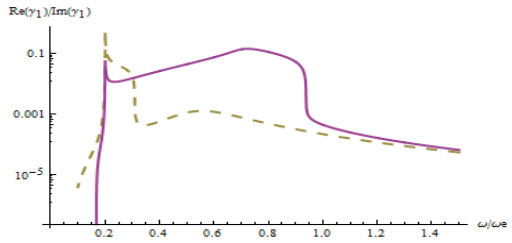


Figure 6. Plot of the $\mathrm{Re}(\gamma_1)/\mathrm{Im}(\gamma_1)$ for TM_0 (dashed line) and TM_3 (solid line) modes.

The frequency region in which $10^{-2} \le \mathrm{Im}(\gamma_1)/\mathrm{Re}(\gamma_1) \le 50$, refer to hybrid modes which is in $0.2\omega_e \le \omega \le 0.3\omega_e$ for TM_0 and $0.2\omega_e \le \omega \le 0.95\omega_e$ for TM_3 modes [7]. Now, we can study the effect of structural parameter on the behavior of modes in the waveguides. Figure 7 illustrates the behavior of TM0 and TM1 modes in metamaterial slab guide by considering different values for F.

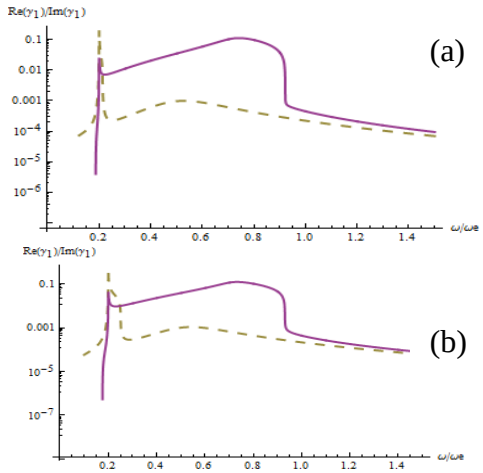


Figure 7. Plot of $\mathrm{Re}(\gamma_1)/\mathrm{Im}(\gamma_1)$ for TM_0 (dashed line) and TM_3 (solid line) for (a) F=0.1 and (b) F=0.3.

Fig.7 shows that the hybrid mode region for TM_0 mode increases by increasing F. This behavior is interesting as by making changes in the metamaterial structures, that can change F, the modes behavior become tunable. This tenability can have applications in the future waveguides construction.

4. Conclusion

We introduced some effective structural parameters, as magnetic oscillation strength and magnetic damping constant, to study their effects on the n of metamaterial and modes behavior in metamaterial-based waveguides.

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