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بهبود شتاب‌دهی الکترون در خلأ با استفاده از برخورد مایل پالس لیزر چیرپ

حسین صابری و بهروز مراغه‌چی

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چکیده- در این پژوهش، ما اندرکنش الکترون با پالس لیزر هرمیت-گاوسی $(0,0)$ را بررسی می‌کنیم. یک پالس لیزر قطبیده خطی و چیرپ را که به طور مایل نسبت به محور z منتشر می‌شود، در نظر می‌گیریم. فرض می‌شود چیرپ فرکانس به صورت خطی با زمان و مکان تغییر می‌کند. الکترون در لحظه اولیه ساکن و در مبدأ در نظر گرفته می‌شود. با استفاده از کد ذره‌ای سه-بعدی و با حل معادلات حرکت نیوتون، حرکت الکترون را به صورت عددی بررسی می‌کنیم. ما پارامتر چیرپ بهینه را برای بیشینه انرژی الکترون و زاویه تابش بهینه را برای کمینه زاویه پراکندگی الکترون به دست می‌آوریم. با این کمیت‌های بهینه، هنگامی که الکترون به انرژی نهایی خود می‌رسد فاصله آن تا محور z با آهنگ بسیار کمی افزایش می‌یابد؛ در نتیجه الکترون در نزدیکی راستای شتاب‌دهی باقی خواهد ماند.

کلیدواژه- شتاب‌دهی الکترون در خلأ، پالس لیزر چیرپ، مد هرمیت-گاوسی $(0,0)$، انتشار مایل

Enhancement of Electron Acceleration in Vacuum by Using an Obliquely Incident Chirped Laser Pulse

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Abstract- We consider a linearly polarized frequency-chirped Hermit-Gaussian $(0,0)$ mode laser propagating obliquely with respect to the z-axis. A linear chirp is assumed in which local frequency varies linearly with time and space. The laser interacts with an electron initially at rest located at the origin. Electron motion is investigated through a numerical simulation using a three-dimensional particle trajectory code by solving the relativistic Newton's equations of motion with corresponding Lorentz force. We find optimum chirp parameter for the maximum electron energy and optimum propagation angle of the laser for the minimum scattering angle of the electron. After the electron attains the final energy, the distance to the z-axis grows slightly when the electron moves along the z-axis which implies that the electron remains restricted near the acceleration direction.

Keywords: Vacuum Electron Acceleration, Chirped Laser Pulse, Hermit-Gaussian $(0,0)$ mode, Oblique Propagation

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1 Introduction

Mechanisms of electron acceleration in vacuum use direct interaction of strong laser fields to accelerate electrons and because of its experimental simplicity has some advantages over laser-plasma accelerators. Furthermore, vacuum electron acceleration by laser works in a single-particle regime without complexities associated with laser-plasma accelerators. Acceleration of initially nonrelativistic electrons [1] and electrons with initial relativistic energies of 20 MeV [2] in vacuum has been observed experimentally by interaction of a high-intensity laser pulse with free electrons.

The Hermit-Gaussian $(0,0)$ mode laser has a spatial and temporal Gaussian shape, so it has an intensity peak at the central axis. Therefore, the electron receives a strong ponderomotive force in the transverse direction and scatters away. [3] Recent studies [4-6] show that the frequency chirping of ultra-short laser pulses can play a crucial role on acceleration of electrons in vacuum. In other words, if instantaneous frequency of the laser changes with time and space, the symmetry between acceleration and deceleration stage will be removed, and the electron could obtain higher energies.

In the following, we investigate the interaction of an electron with a linearly polarized chirped laser pulse propagating obliquely with respect to the z-axis. The electron is assumed to be initially at rest at the origin. By numerical investigations, we will find the optimum chirp parameter for maximum electron energy and the optimum propagation angle of the laser for minimum scattering angle of the electron.

2 Acceleration Equations

In our simulation, we consider a linearly polarized Hermit-Gaussian $(0,0)$ laser pulse with spatial and temporal Gaussian profile. Moreover, a linear frequency chirping is assumed. A linearly polarized frequency-chirped laser pulse is more effective for single electron acceleration. [6]

A linearly chirped laser pulse propagating along direction $\hat{e}_{z'}=\hat{e}_x\sin\theta+\hat{e}_z\cos\theta$ interacts with an electron initially at rest located at the origin. Here, θ is the incident angle that the propagation direction makes with the z axis. It is assumed that the laser pulse is linearly polarized in the transverse direction x' with a longitudinal component in the z' direction in the rotated reference frame at angle θ. In other words, $\mathbf{E}=E_{x'}\hat{e}_{x'}+E_{z'}\hat{e}_{z'}$ where $\hat{e}_{x'}$ and $\hat{e}_{z'}$ are unit vectors. In the paraxial approximation, the transverse electric field component of the laser is given by [5, 7]

$$E_{x'}=E_0\frac{W_0}{W(z')}\exp\left(-\frac{r'^2}{W^2(z')}\right)\exp\left(-\frac{\xi'^2}{\tau^2}\right)\times\cos\left(\omega(\xi')\xi'+\frac{k(\xi')r'^2}{2R(z')}-\varphi(z')+\varphi_0\right), \quad (1)$$

where E_0 is the maximum field amplitude, $\omega(\xi')$ is the instantaneous frequency, $\xi'=(z'-z'_L)/c-t$ is the retarded time, z'_L is the pulse peak initial position, τ is the laser pulse duration, φ_0 is the initial phase of the laser, and $r'=\sqrt{x'^2+y'^2}$. The laser spot size, the radius of curvature and the Gouy phase are

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$$W(z')=W_0\sqrt{1+(z'/z'_R)^2}, \quad (2)$$

$$R(z')=z'+z'^2_R/z', \quad (3)$$

$$\varphi(z')=\tan^{-1}(z'/z'_R), \quad (4)$$

where W_0 is the minimum spot size (waist), $z'_R=k(\xi')W_0^2/2$ is the Rayleigh length, $k(\xi')=\omega(\xi')/c$ is the wave number, $\omega(\xi')=\omega_0(1+\alpha\xi')$ is the instantaneous frequency with a linear chirp, ω_0 is the frequency at $\xi'=0$ and α is the chirp parameter. We use the Maxwell's equations in vacuum $\nabla'.\mathbf{E}=0$ and $\partial\mathbf{B}/\partial t=-c\nabla'\times\mathbf{E}$ to find the longitudinal electric field component and the magnetic field components of the laser pulse. The following equations give the coordinate components:

$$x'=x\cos\theta-z\sin\theta,\ \ y'=y,\ \ z'=x\sin\theta+z\cos\theta. \quad (5)$$

The electron motion in such an electromagnetic field is governed by the relativistic Lorentz equation together with an energy gain equation

$$d\mathbf{P}/dt=-e(\mathbf{E}+\mathbf{v}/c\times\mathbf{B}), \quad (6)$$

$$d\left(\gamma m_e c^2\right)\Big/dt=-e\mathbf{v}\cdot\mathbf{E}. \quad (7)$$

The trajectory of the electron is determined by

$$\mathbf{r}(t)=\mathbf{r}_0+\int_0^t\mathbf{v}dt'. \quad (8)$$

Here, $\mathbf{P}=\gamma m_e\mathbf{v}$ is the electron momentum, $\mathbf{v}$ is the electron velocity, and m_e is the electron rest mass. The relativistic factor γ is defined as $\gamma=\left(1-\beta^2\right)^{-1/2}$ where $\beta=|\boldsymbol{\beta}|$ with $\boldsymbol{\beta}=\mathbf{v}/c$, and c is the speed of light in vacuum. e is the absolute value of the charge of the electron and $\mathbf{r}_0$ is the electron's initial position. To simply solve Equations (6)-(8) numerically some normalization has been done as follows: The spatial coordinates are normalized by c/ω_0, time is normalized as $\omega_0 t$, and the momentum is normalized as $\mathbf{p}=\mathbf{P}/m_e c$. The electric and magnetic fields are normalized by $m_e c\omega_0/e$ such that $a_0=eE_0/m_e c\omega_0$. The chirp parameter is normalized by ω_0^2.

The acceleration scheme proposed herein is analyzed in the framework of a relativistic three-dimensional single particle theory by solving the relativistic Newton's equations of motion with corresponding Lorentz force. These equations are solved numerically using the fourth-order Runge-Kutta method.

3 Numerical Simulation

In this section, we present the simulation of the electron dynamics in our proposed scheme. The common laser parameters are $a_0=3$, $\lambda_0=1\mu m$, $\tau_0=100\,fs$, and $W_0=10\mu m$. The parameters are chosen to be similar to those of reference 1. The laser pulse is sufficiently far away from the electron at $t=0$ such that the field amplitude at the initial position of the electron can be neglected. A laser peak pulse position of $z'_L=-100\mu m$ is sufficient. The electron is assumed to be initially at rest located at the origin.

Due to the ponderomotive force, an electron subjected to a high-intensity laser pulse could attain a transverse momentum and scatter away from the pulse focus to reach high energies.[3] The scattering angle with respect to the z-axis could be obtained from $\tan^{-1}(p_\perp/p_z)$ where $p_\perp$ and p_z are the final transverse and longitudinal momenta of the electron. In the case of Hermit-Gaussian$(0,0)$ mode without frequency chirping the electron achieves final energy $\gamma=2.7$ with scattering angle of 44.2°.

By using a frequency-chirped laser with proper chirp parameter, the electron could achieve higher energies with lower scattering angles. Figure 1(a) shows the plot of the electron final energy versus chirp parameter α of the laser pulse propagating along the z-direction. Optimum chirp parameters to maximize electron final energy are $\alpha=-0.013$ and $\alpha=+0.01$. Figure 1(b) shows the scattering angle of the electron versus chirp parameter. It can be seen that with $\alpha<0$ the electron could achieve higher energies with lower scattering angles. With the chirp parameter $\alpha=-0.013$ the electron achieves final energy $\gamma=204$ with scattering angle of 4.41° and with $\alpha=+0.01$ the electron achieves final energy $\gamma=175$ with scattering angle of 5.62°.

If the laser pulse propagates obliquely with respect to the z-axis at a specified angle, the scattering angle could also be minimized for a given chirp parameter. However, the final energy is also the same as Fig. 1(a). In Fig. 2 the initial propagation angle of the laser pulse, for which the scattering

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angle is minimized, is shown. The minimum scattering angle is also plotted.

In the interaction of a chirped laser pulse, an electron receives a transverse momentum gain in the direction of the polarization. The increase in the transverse momentum results in an increase in the longitudinal momentum due to an increase in longitudinal $\mathbf{v}\times\mathbf{B}$ force. [4, 5]

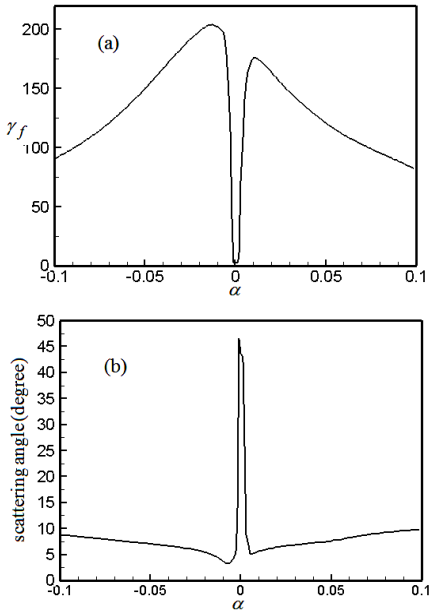


Figure 1: Electron final energy (a) and scattering angle (b) vs. laser chirp parameter α .

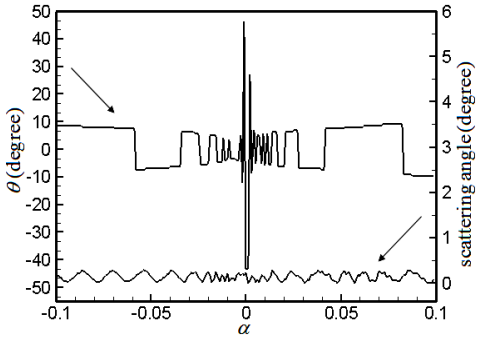


Figure 3: Initial laser propagation angle, for which the scattering angle is a minimum, vs. chirp parameter. The minimum scattering angle is also plotted.

Figure 3(a) shows the dimensionless transverse momentum of a rest electron accelerated by a chirped laser pulse propagating at angle $\theta=-4.58^\circ$ (solid line) and $\theta=0$ (dashed line). For the propagation angle $\theta=-4.58^\circ$, the scattering angle will be minimized for chirp parameter of $\alpha=-0.013$. It can be seen that in the case of oblique interaction, the absolute value of the final momentum is smaller than that of interaction with a laser pulse propagating along the z-direction, which implies that the electron remains restricted near the acceleration direction and a large transverse momentum is avoided. So, as can be seen in Fig. 3(b), after the electron attains the final energy, the distance to the z-axis grows slightly when the electron moves along the z-axis. The electron scattering angle will be 0.17° which is smaller than 4.4° for acceleration by a pulse propagating along the z-axis.

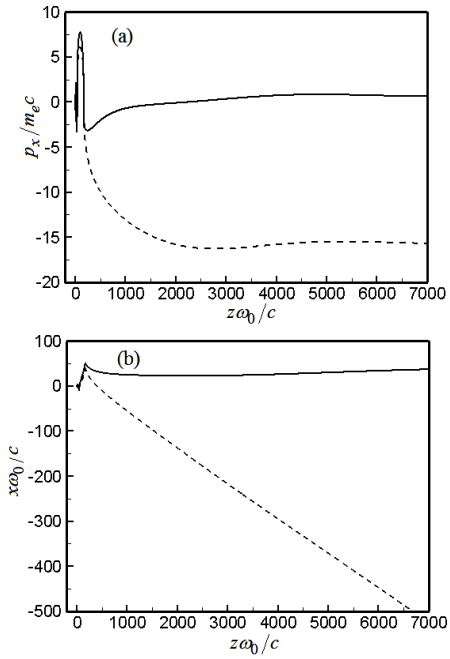


Figure 4: Electron dimensionless transverse momentum (a) and its trajectory in the dimensionless x-z plane (b).

4 CONCLUSIONS

We found that the electron acceleration could be enhanced by using a chirped laser propagating at a specified angle with respect to the z-axis. By numerical investigations, we found the optimum chirp parameter for maximum electron energy and the optimum propagation angle of the laser for minimum scattering angle of the electron.

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