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ساختارهای نوسانی و آشوبناک در کاواک های تحت شفافیت القایی الکترومغناطیسی

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شفافیت القایی الکترومغناطیسی بعنوان یکی از پدیده های اپتیکی کوانتومی زمینه های تحقیقاتی بسیار جذابی را بوجود آورده است. این مکانیسم که باعث می شود محیط نسبت به نور فرودی شفاف شده و جذب در نزدیکی تشدید به صفر برسد، برای کاربردهای مختلفی در زمینه انتقال، پردازش و ذخیره سازی نوری اطلاعات حائز اهمیت شده است. همچنین، اخیرا نشان داده شده است که کاواک های حاوی چنین محیط هایی از دینامیک پیچیده و غیرخطی بالایی برخوردار هستند که مطالعات بیشتری را می طلبد. ما با استفاده از معادله میدان میانگین برای توصیف کاواک غیرخطی حاوی اتم های سه ترازی تحت شفافیت القایی الکترومغناطیسی و نظر گرفتن رژیم واکانونی، دینامیک فضایی-زمانی سیستم را شبیه سازی کرده و به مطالعه جواب های عرضی خاص آن پرداخته ایم. نشان می دهیم که طرحواره ها و سالیتون های کاواک (تاریک) تشکیل شده در این رژیم، با تغییر پارامتر کنترلی نوسان های منظم و به تدریج آشوبناک از خود نشان می دهند.

کلید واژه– آشوب، خود واکانونی، سالیتون، شفافیت القایی الکترومغناطیسی، طرحواره فضایی

Oscillating and chaotic structures in EIT cavities

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Abstract-Spatially periodic and localized structures in the transverse plane of a medium displaying electromagnetically induced transparency in an optical cavity and under the action of two pumps are investigated. The system supports a multitude of different complex spatial structures depending on the chosen initial condition. We explore regimes of multistable patterns, filaments, stable defects, scrolling structures, nested patterns, fronts, and the spontaneous occurrence of multiple cavity solitons.

Keywords: EIT, Solitons, Patterns, Chaos, self-defocusing

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1. Introduction

Electromagnetically induced transparency (EIT) is an example of coherent multi-level processes and has opened a promising window for realization of schemes needed for quantum information systems, coherent control of atomic populations, and mediation of interactions between optical fields. The majority of EIT studies have been carried out in gaseous media where the dominant broadening mechanism is that of a homogenous type leading to a variety of applications including slow light propagation, optical storage, precision measurements, amplification and lasing without inversion [1]. Complex spatial structures in the output of an optical cavity containing a medium close to EIT have shapes and stability with strong dependence on parameter values and initial conditions. It has been shown that sensitivity to initial conditions in the final evolution of this system is due to a generalized multistability. The multistability and the nature of the stable states are in turn affected by relatively small changes in the parameter values since different branches of solutions experience different sequence of bifurcations. For some initial conditions, a given set of expected spatially periodic solutions is attained. For other initial conditions, however, the spatiotemporal evolution moves the system in different directions in the phase space to either coexistent regions of different patterns or to stable defects surrounded by regions of different orientation of a single transverse pattern [2-4].

After the early observations of regular and stationary patterns and localized structures, the introduction of oscillatory patterns and their analysis via secondary bifurcations paved the way for understanding the possible routes for chaos and symmetry breaking mechanisms [5-7]. It has been reported in [5] that the interplay between space and time leads to a series of bifurcations showing spatial-period multiplying quasi-periodicity for hexagonal patterns. It has also been shown that spatial order is completely lost through series of instabilities which cause the system to enter a regime of optical turbulence. While their studies were based on a Kerr cavity with self-focusing nonlinearity, in another study [6] they have found strong correlation between intensity fluctuations of any arbitrary pair of wave vectors of the pattern.

Here, we focus on the dynamical behaviour of such transverse solutions under self-defocusing nonlinearity. We use EIT features in a 3-level Λ atomic system confined in a cavity and show that through proper choice of parameter values, the system exhibits bistability, Turing instability, branches of transverse solutions and Dark Cavity Solitons (DCSs). We particularly show that dark hexagonal patterns experience a bifurcation which gives them regular and then chaotic oscillations. We also show that oscillating dark cavity solitons can form in a specific region where chaotic patterns coexist with homogenous stationary solutions. These specific dynamics are studied via time traces and argand plane trajectories along with power spectrum analyses to further elucidate the chaotic transitions.

The paper is organized as follows: The model, associated equations, homogenous stationary solutions and linear stability analysis are discussed in Sec. II. Section III describes the variety of solutions obtained by numerical simulations and their dynamical features. We then draw conclusions in Sec IV.

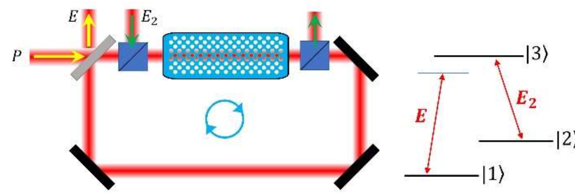

Fig.1 The cavity configuration under the action of the pump P and coupling E_2 fields, along with the Λ atomic scheme with two ground states $|1\rangle$, $|2\rangle$ and a single excited level $|3\rangle$.

2. The Model

The system of interest is a ring cavity filled with three-level atomic vapor (for example, Rb atoms) in a Λ configuration and under the action of two optical pumps [4]. The schematic representation of the cavity and the configuration of atomic vapor are shown in Fig.1(a). The injected field P is detuned by Δ from the resonance of the atomic transition $|3\rangle \rightarrow |1\rangle$ while the coupling beam E_2 is kept at resonance with the transition $|3\rangle \rightarrow |2\rangle$. The cavity mirrors resonate the field E which is detuned from the pump by θ. In the present model, the field E_2 is not resonated in the cavity, which is realistic if the atomic frequencies are well separated. The mean-field equation for a beam propagating in the medium inside the optical cavity of Fig. 1 is:

$$\partial_t E = P - (1 + i\theta)E + 2iC\rho_{13} + i\nabla^2 E \qquad (1)$$

C is the cooperative parameter directly proportional to the atomic density n_a through

$$2C = \frac{n_a \mu^2 kL}{2\hbar\gamma\varepsilon_0 T} \qquad (2)$$

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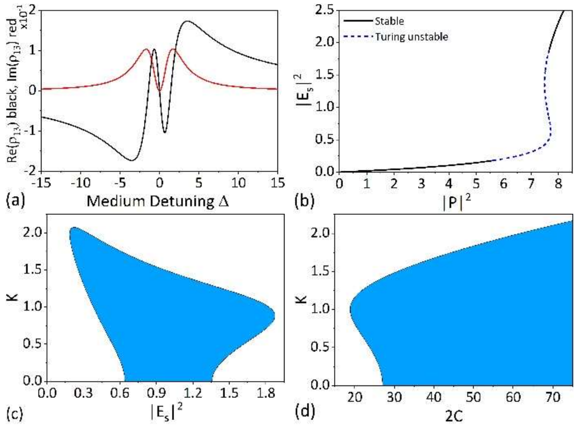

Fig. 2. (a) The imaginary (red) and real (black) parts of the complex susceptibility χ for $|E_s|^2=|E_2|^2=1$. (b) Bistability in the input and output intensities and (c) Turing instability domain responsible for pattern formation for fixed values of $\Delta = -0.2$, $\theta = -1$, $2C = 30$, and $|E_2|^2 = 1$. (d) Map of (c) on the 2C−K space for fixed value of $|E_s|^2 = 1$.

where μ is the atomic transition dipole moment, k is the wave number of the field, L is the length of the cavity, γ is the atomic linewidth, ε_0 is the permittivity of free space, and T is the cavity mirror transmittivity. ρ_{13} is the off-diagonal density matrix element proportional to the field amplitude E and the complex susceptibility χ:

$$\rho_{13} = \chi E = -\frac{\Delta|E_2|^2(|E_2|^2+|E|^2-i\Delta)}{(|E_2|^2+|E|^2)^3}E \tag{3}$$

The real (dispersion) and imaginary (absorption) parts of Eq. (3) are shown in Fig. 2(a) in terms of material detuning where the vanishing absorption close to the medium resonance clearly evidences the EIT phenomenon.

The diffraction term is given by the Laplacian operator in two transverse dimensions and time is normalized to the photon lifetime in the cavity. Split-step programming is adopted for simulation of the system which consists in a Runge-Kutta algorithm for the time evolution and Fast Fourier Transform (FFT) for dealing with the diffraction term. We assumed a box with 64×64 grid points. In this paper, we turn our attention to parameter values leading to self-defocusing nonlinearity where negative hexagons and DCSs are expected to form. In Fig. 2(b)-(d), the bistability curve for this regime is shown along with the Turing instability domains with respect to the stationary intensity $|E_s|^2$ and 2C calculated via linear stability analysis. It is seen that the lower intensity branch of the bistability curve is affected by the Turing instability which confirms the possibility of DCS formation.

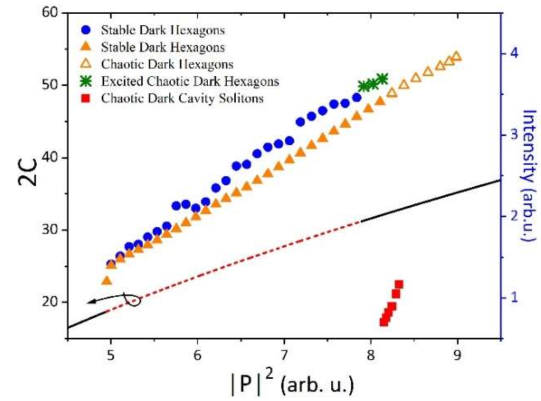

Fig. 3. 2C versus pump intensity and corresponding pattern intensities.

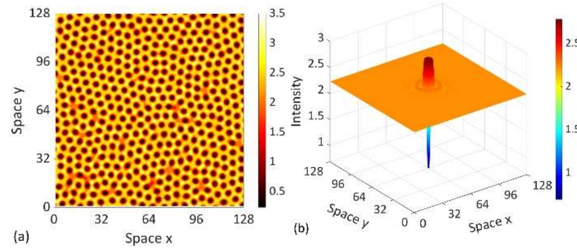

Fig. 4. Snapshots from simulation. (a) Hπ for 2C=31.5 and $|P|^2 = 7.70$. (b) Dark cavity soliton for 2C=32.8 and $|P|^2 = 8.32$.

For the regime considered in this paper, sensitivity to initial conditions contributes in having two branches of solutions in the Turing unstable region with different properties. By choosing the initial value of the control parameter 2C less than the bifurcation point (2C=18.8) and adiabatically following the increase of 2C value, it is possible to catch a secondary bifurcation point at 2C=32.50 beyond which the stable negative hexagons start to oscillate and eventually we achieve a regime of chaotic Hπ. However, by arbitrarily choosing the initial value of the control parameter in the Turing interval and simulating each point individually a sequence of stable negative hexagons are obtained with no evidence of oscillations. These solutions are shown in Fig. 3. Examples of these pattern solutions and DCS are shown in Fig. 4.

3. Oscillating and Chaotic Negative Hexagons

For the case of self-defocusing nonlinearity here, adiabatic scan results in a wider range of pattern solutions compared to the solutions obtained via individual simulation of 2C values inside the Turing interval. In fact, the interval for pattern solutions in the case of adiabatic scan extends from 2C=18.8 up to 35.15 while it is just up to 31.25 for the branch obtained by individual simulations. Moreover, a secondary bifurcation point is reached for transition

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of stable negative hexagons to oscillating and eventually chaotic negative hexagons. As it is depicted in Fig. 3, part of the branch that corresponds to chaotic hexagons in adiabatic scan is bistable with homogeneous solutions. The interval begins from 2C=31.25 and ends at 32.80. This implies that one can excite chaotic hexagons from homogeneous solutions by transient switching pulses of appropriate width which is possible in the parameter range of 2C=31.25-32.10.

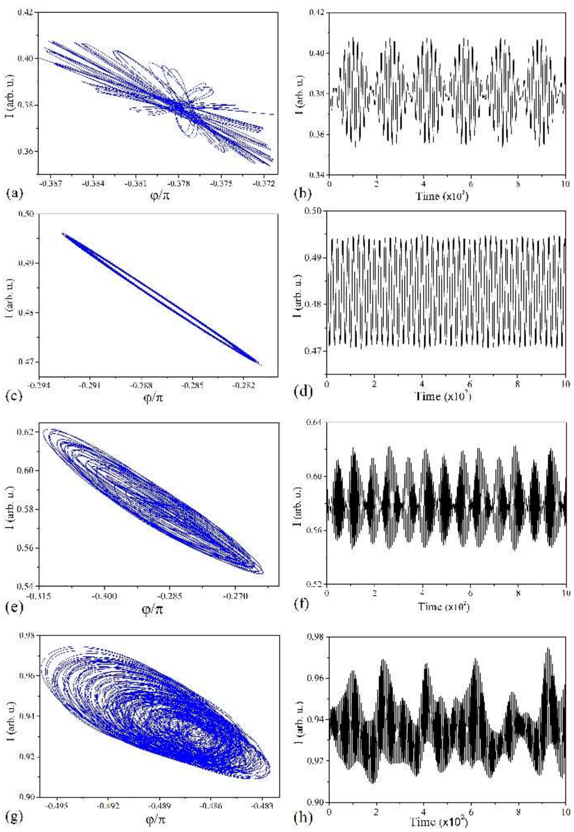

Fig.5. Left column shows the projection of the time evolution of excited chaotic hexagons onto the 2D phase space corresponding to the phase and intensity of the central point of the patterns. Panels on the right depict the time trace of the associated intensity oscillations. For (a,b) $2C$ is 31.30, for (c,d) it is 31.50, 31.70 for (e,f) and 31.90 for (g,h).

In Fig. 5 the intensity time trace of the maximum point of the hexagonal patterns are shown for different 2C values along with their intensity-phase sub-space trajectories and power spectra in Fig. 6. Transition to chaos is evident from the break up of the limit cycle in Fig. 6(a) to Fig. 6(g) by increasing the control parameter value from 2C=31.30 to 31.90. This is consistent with the changes happening in the power spectra from Fig. 6(a) to Fig. 6(d). Transition to chaos is better understood by noting that the number of frequency components included in the oscillations starts from two frequencies and increases to many components around the central peaks.

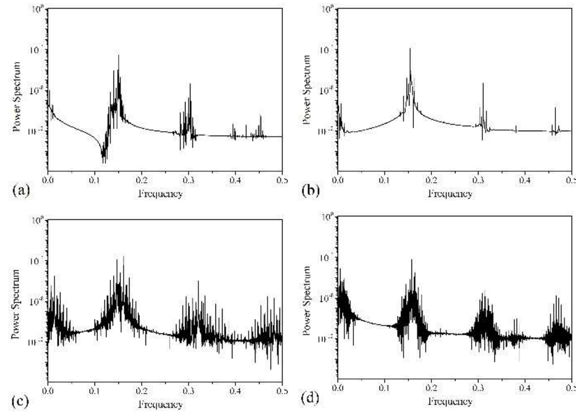

Fig. 6. Power spectrum for oscillations of excited dark hexagons in 2C = 31.30 (a), 2C = 31.50 (b), 2C = 31.70 (c) and 2C = 31.90 (d).

4. Conclusions

By considering self-defocusing regime in a cavity containing 3-level atoms which displays EIT, the transition from homogeneous state to stationary and oscillating dark hexagonal patterns is studied. It is shown that by increasing the value of the control parameter, the repetitive character of trajectories in the sub-space made up of intensity and phase of the central point of the hexagons breaks to a random one indicating a route for chaos. As it is depicted in Fig. 3, this route for chaos also affects the cavity solitons branch giving them oscillating dynamics. These oscillating extended and localized structures are an interesting and hybrid product of multi-level quantum coherent phenomenon (EIT) and nonlinear dynamics in a complex system and can be of interest in optical processing and communication applications.

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