



## دینامیک سولیتونی جاذب اشباع پذیر حاکم بر نانو ذرات کلوییدی

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چکیده - این مقاله یک حل ساده اما جامع را برای معادله هاوس - که برای توصیف دینامیک قفل زنی مد بکار می رود - به منظور مدلسازی جاذب اشباع پذیر در نانوذرات کلوییدی ارائه می کند. تشکیل سولیتون بعنوان ساز و کار اصلی شکل گیری پالس فرض گردیده است. مدل گاز غیر ایده آل برای نانوذرات بکار گرفته شده است. فرایند جاذب اشباع پذیر به شکل قفل زنی انفعالی شناسایی می شود. نتایج حاصل از مدل توافق خوبی با مشاهدات تجربی دارد؛ طی این مشاهدات، نانوذرات کلوییدی محیط های مناسبی برای هدف استفاده به عنوان جاذب اشباع پذیر معرفی شده اند.

کلید واژه - جاذب اشباع پذیر، سولیتون، قفل زنی انفعالی، نانوذرات کلوییدی

## Soliton Dynamics of Saturable Absorber in Colloidal Nanoparticles

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Abstract- This paper presents a simple, but comprehensive solution of Haus equation - to describe the dynamics of mode-locking- to model Saturable Absorption in colloidal nanoparticles. Soliton formation is considered as the main mechanism of pulse shaping. Non-ideal gas model for nanosuspensions nonlinearity is utilized. Saturable Absorber procedure is detected as passive mode-locking. The results show a good matching with the experimental observations in which the nanosuspensions have introduced as suitable media for Saturable Absorption.

Keywords: Saturable Absorption, Soliton, Passive mode-locking, Colloidal nanoparticles

## 1 Introduction

Nanoparticles and Colloidal nanosuspensions are well-known for their high nonlinear response even at optically low intensities; their high performance of controlling light with light predicts the promising future of advances in all-optical devices and systems. Recently, Alexander Shamray and his group have experimentally shown that the optical bistability in colloidal solution of  $\text{LaF}_3\text{:Er,Yb}$  nanocrystals is truly attainable at low optical power [1]. In spite of multiple studies on nanosuspensions nonlinearity and Saturable Absorption (SA) via nanosuspensions [2-5], there seems to be a lack of theoretical model based on intracavity dominating equations to describe the dynamics of SA through these media.

## 2 Theoretical description of problem

Theoretical dynamics of pulse-shaping can be described by the Haus equation of mode-locking [6]:

$$T_R \frac{\partial A(T, t)}{\partial T} = -iD \frac{\partial^2 A}{\partial t^2} + i\gamma |A|^2 A + \left[ g - l + D_g \frac{\partial^2}{\partial t^2} - q(T, t) \right] A(T, t) \quad (1)$$

Where,  $A(T, t)$  is slowly varying field envelop,  $T$  is time scaled to one cavity round trip  $T_R$ ,  $t$  is time scaled to pulse duration,  $D$  is Group Velocity Dispersion (GVD) and  $D_g$  is gain dispersion;  $\gamma = \frac{2\pi}{\lambda_0} n_2$  is the nonlinear coefficient required for Self Phase Modulation (SPM) whereas  $\lambda_0$  is the center wavelength of the pulse,  $n_2$  is the nonlinear refractive index of nanosuspension;  $g$  and  $l$  are the gain and loss of the cavity respectively;  $q(T, t)$  is the response of the absorber. For simplicity,  $g - l$  is substituted by the quantity  $NET$  gain. It should be noted that intracavity loss  $l$  is quite different from the loss due to absorption of the nonlinear media; the latter is postponed to be discussed. The corresponding pulse energy can be written as:

$$E = \int_{-T_R/2}^{T_R/2} |A(T, t)|^2 dt \quad (2)$$

### 2.1 Soliton Regime

Soliton formation is considered as the dominant pulse-shaping mechanism. The function to describe the pulse shape is given below:

$$f(t) = \sqrt{\frac{1}{2\tau}} \text{sech}\left(\frac{t}{\tau}\right) \quad (3)$$

$\tau$  is pulse width which is defined as the full-width at half-maximum (FWHM) of the soliton. Soliton formation is a consequence of the balance between GVD and SPM inside the cavity; this in turn leads

to the equation  $\frac{|D|}{\tau^2} = \frac{\gamma E}{4\tau}$ . The equation can be written as the pulse width related to the energy of the pulse:

$$\tau = \frac{4|D|}{\gamma E} \quad (4)$$

On the other hand, energy rate equation is proved to be written as [6]:

$$T_R \frac{\partial E}{\partial T} = 2g_{NET} - 2q(E) \quad (5)$$

Where:

$$g_{NET} = \frac{g_{0,NET}}{1 + \frac{E}{E_s}} \quad (6)$$

Through the steady state, the ratio between intracavity GVD and gain dispersion is set to be at a certain value [6].

It is inevitable to introduce TPA (Two Photon Absorption) loss through the calculations in order to measure Minimum Pulse Energy (MPE) needed for the desired case of single pulse mode locking. Equation describing MPE is obtained according to [6]:

$$E_{\min,SSA} = \sqrt[3]{\frac{q_M E_s}{\alpha_{TPA}}} \quad (7)$$

Here,  $q_M$  is the maximum absorber response,

$\alpha_{TPA} \propto \frac{\alpha\gamma}{12D}$  is the loss pertained to TPA. The quantity  $\alpha$  is the aforementioned absorption of nonlinear medium due to Rayleigh scattering.

### 2.2 Non-ideal gas nonlinearity model of nanosuspensions

According to this approach, the first order nonlinear refractive indices of  $n_2$  is yielded as:

$$n_2 = \left( \frac{n_{2k}}{1 + 2(B_2/V_p)f_0 + 3(B_3/V_p^2)f_0^2} \right) \quad (8)$$

In these equations,  $n_{2k}$  is artificial Kerr nonlinearity,  $B_2$  and  $B_3$  are second and third

Virial coefficients,  $V_p$  is the volume of nanoparticle,  $f_0$  is the field free background filling factor of the colloidal nanoparticles.

The first nonlinear loss of TPA due to Rayleigh scattering is given by

$$\alpha = \frac{1}{2} \frac{\sigma \rho_0}{I_c (1 + 2(B_2/V_p)f_0 + 3(B_3/V_p^2)f_0^2)} \quad \text{where } \rho_0 \text{ is}$$

the initial nanoparticle density inside the colloid,  $I_c$  pertains to the light intensity keeping the limit of exponential model for nonlinearity and

$$\sigma = \frac{128\pi^5 a^2 n_b^4}{3} \left( \frac{a}{\lambda_0} \right)^4 \left( \frac{m^2 - 1}{m^2 + 2} \right)^2$$

is scattering cross section;  $n_b$  is background refractive index of colloid and  $m = n_p / n_b$  is the ratio of nanoparticles refractive index to  $n_b$ .

### 3 Analytical Results and Discussion

For a sinusoidal response of the absorber[8], one pulse operation is assumed. Equation (9) performs a simple but comprehensive analytical solution of the equation (5) to describe the dynamics of soliton regime inside mode-locked cavity for successive cavity round trips. Once the energy inside the cavity is increased, SA response will be strengthened and the intracavity energy will be diminished to its primary value. This implies that the field envelop inside the cavity is passively mode-locked.

$$E(T) = \frac{1}{2} \frac{g_{0,NET}}{1 + \frac{E(T - T_R)}{E_s}} \times \alpha_{TPA} (1 - q_M \cos(\pi E(T - T_R))) E(T - T_R) \quad (9)$$

The iterative form of equation (9) explicates the state of the system as a function of  $q_M$  and  $E_s$ . For sufficiently small values of  $q_M$  and  $E_s$ , cw stable operation will be provided if the gain of the medium is adequate. To ensure that high efficient laser cavity is attainable,  $q_M$  should be increased; afterward, above a certain threshold value of  $E_s$ , the system transforms to mode-locking state. Figure 1 shows system transition from the primary cw state toward the chaotic state as  $q_M$  and  $E_s$  are increased.

#### 3.1 Effect of Nanoparticle characteristics on the system state

Experimental measured data are taken from Majles

Ara, *et al.*[3]. Calculations based on the presented model show that for the nanoparticles density order of 10 times larger than what is assumed, steady state mode-locking will be vanished and the system will experience a fully chaotic regime. Conversely, for 10 times lower order steady state mode-locking regime will no more be available.

#### 3.2 MPE required for passively mode-locking

Equation (7) stands for MPE required for steady state mod-locking. Figure 2.a shows the dependence of MPE on the average diameter of nanoparticles. It can be seen that there is an optimum point in which the MPE is increased over/under the value. The procedure offers to use small sizes of nano-scaled particles. Indebted to the large values of  $\alpha$  and  $\gamma$  pertained to nanosuspensions, researchers believe/experience that passive mode-locking is attainable at extremely low powers [4-5]. Infinitesimal experimental reported values of  $D$  inside nanosuspensions intensify this opinion (refer to equation (8)).

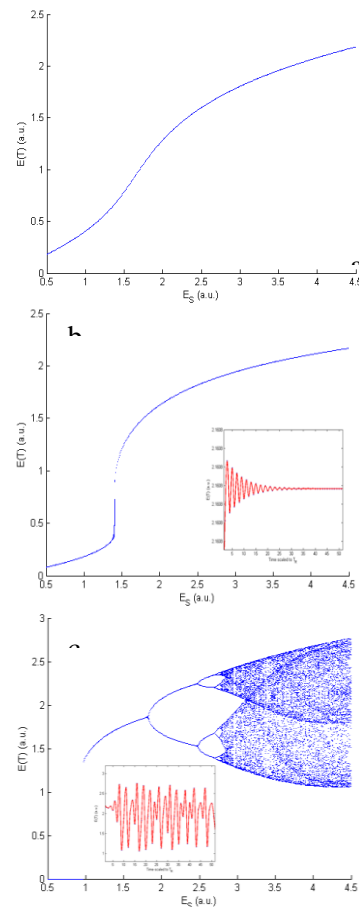


Figure 1: All available states of the system inside the cavity as a function of  $E_s$ , a) cw steady state,  $q_M = 0.2$ , b) Bistable state (Inset shows how the system

transforms to upper stable state),  $q_M = 0.3$ , c) Chaotic state (Inset shows how the system has instability over time),  $q_M = 0.6$ .

## 4 Conclusion

A simple, but comprehensive solution of Haus equation is presented. Using the Non-ideal gas model of nonlinearity for colloidal nanoparticles, a model is constructed to investigate Saturable Absorption inside nanosuspensions. These results seem to be truly matched with experimental observations.

nanoparticles", *Optical Materials*, Vol. 35, pp. 1431-1435, 2013.

- [4] Leonardo De Boni , Erin Leigh Wood , Carlos Toro , Florencio E. Hernandez, "Optical Saturable Absorption in Gold Nanoparticles", *Plasmonics*, Vol.3, pp. 171-176, 2008
- [5] Misha Hari , S. Mathew, B. Nithyaja, Santhi Ani Joseph, V. P. N. Nampoorei, P. Radhakrishnan, "Saturable and reverse saturable absorption in aqueous silver nanoparticles at off-resonant wavelength", *Opt Quant Electron*, Vol.43, pp. 49-58, 2012

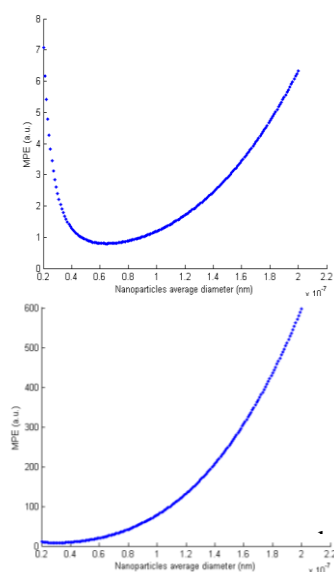


Figure 2: MPE vs. nanoparticles size a)for assumed nanoparticles density b) for 10 times higher nanoparticles density

## References

- [1] George Pobegalov, Peter Agruzov, Igor Ilichev, and Alexander Shamray, "Low-power optical bistability and hysteresis in the laser systems with absorbing nanosuspension", *Optics Letters*, Vol. 39, No. 10, pp. 2819-2822, 2014.
- [2] N. M. R. Peres, Yu. V. Bludov, Jaime E. Santos, Antti-Pekka Jauho, and M. I. Vasilevskiy, "Optical bistability of graphene in the terahertz", *Phys Rev B*, 90, pp. 125425-1-7, 2014
- [3] M. H. Majles Ara, E.Koushki, H. Akherat Doost, "Optical bistability in TiO<sub>2</sub>